MATHEMATICS

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AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XI (PQRS)

ARITHMETIC PROGRESSIONS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. If $a_1, a_2, a_3, ..., a_n, ...$ is a sequence, then the expression $a_1 + a_2 + a_3 + ... + a_n + ...$ is called a series. A series is called a finite series if it has got finite number of terms, otherwise, it is called an infinite series.
2. A sequence is called an arithmetic progression if the difference of a term and the previous term is always same, i.e.

   $a_{n+1} - a_n$ = constant (= d) for all $n \in N$

   The constant difference 'd' is called the common difference.
3. A sequence is an arithmetic progression if and only if its nth terms is a linear expression in n and in such a case the common difference is equal to the co–efficient of n.
4. If a is the first term and d is the common difference of an A.p., then its nth term is given by

   $a_n = a + (n - 1)\, d$
5. The following ways of selecting terms of an A.P. are generally very convenient :

Number of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

6. The sum S_n of n terms of an A.P. with first term 'a' and common difference 'd' is given by

   $$S_n = \frac{n}{2}\{2a + (n - 1)\, d\}$$

   or, $S_n = \frac{n}{2}\{a + l\}$, where l = last term = $a + (n - 1)\, d$.
7. If the sum S_n of n terms of a sequence is given, then n^{th} term a_n of the sequence can be determined by using the formula :

   $a_n = S_n - S_{n-1}$
8. If the ratio of the sums of n terms of two A.P.'s is given, then the ratio of their n^{th} terms is obtained by replacing n by (2n – 1) in the given ratio.
9. Three numbers a, b, c are in A.P. iff $2b = a + c$
10. If a, b, c are in A.P., then b is called the arithmetic mean of a and c.
11. The arithmetic mean of a and b is $\frac{a+b}{2}$.
12. If n numbers $A_1, A_2, ..., A_n$ are inserted between two given numbers a and b such that $a, A_1, A_2, ..., A_n, b$ is an arithmetic progression, then $A_1, A_2, ..., A_n$ are known as n arithmetic means between a and b and the common difference of the A.P. is

    $$d = \frac{b-a}{n+1}$$

    Also, $$A_1 + A_2 + ... + A_n = n\left(\frac{a+b}{2}\right).$$

13. In an A.P. the sum of the terms equidistant from the beginning and the end is always same and is equal to the sum of first and last term.

12. A sequence is an A.P. iff the sum of its n terms is of the form $An^2 + Bn$ i.e. a quadratic expression in n and in such a case the common difference is twice the coefficient of n^2.

EXERCISE-1

1. Show that the sequence $< a_n >$ defined by $a_n = 4n + 5$ is an A.P. Also, find its common difference.
2. The n^{th} term of a sequence is $3n - 2$. Is the sequence an A.P. ? If so, find its term.
3. If the n^{th} term a_n of a sequence is given by $a_n = n^2 - n + 1$, write down its first five terms.
4. The n^{th} term of a sequence is given by $a_n = 2n + 7$. Show that it is an A.P. Also, find its 7th term.
5. Show that the sequence 9, 12, 15, 18, ... is an A.P. Find its 16th term and the general term.
6. Which term of the sequence 4, 9, 14, 19, ... is 124 ?
7. Which term of the sequence $8 - 6i$, $6 - 2i$, ... is (i) purely real (ii) purely imaginary ?
8. If the mth term of an A.P. be 1/n and nth term be 1/m, then show that its (mn)th term is 1.
9. In the arithmetic progressions 2, 5, 8, ... upto 50 terms, and 3, 5, 7, 9, ... upto 60 terms, find how many terms are identical.

10. If $a_1, a_2, a_3, ..., a_n$ be an A.P. of non–zero terms, prove that $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + ... + \frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}$

11. (i) Is 68 a term of the A.P. 7, 10, 13, ... ?
    (ii) Is 302 a term of the A.P. 3, 8, 13, ... ?
12. The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.
13. In a certain A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.
14. How many numbers of two digit are divisible by 3 ?
15. Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7 : 15.
16. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.
17. If the sum of three numbers in A.P. is 24 and their product is 440, find the numbers.
18. Find the sum of all odd integers between 2 and 100 divisible by 3.
19. If the sum of n terms of an A.P. is $pn + qn^2$, where p and q are constants, find the common difference.
20. Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7th term is two more than thrice of its 3rd term.

21. Find the number of terms in the series $20, 19\frac{1}{3}, 18\frac{2}{3}, ...$ of which the sum is 300, explain the double answer.

22. The pth term of an A.P. is a and qth term is b. Prove that the sum of its (p + q) terms is $\frac{p+q}{2}\left\{a + b + \frac{a-b}{p-q}\right\}$.

23. The ratio of the sum of n terms of two A.P.'s is (7n + 1) : (4n + 27). Find the ratio of their mth terms.

24. Let S_n be the sum of first n terms of an A.P. with non–zero common difference. Find the ratio of first term and common difference if $\frac{S_{n_1 n_2}}{S_{n_1}}$ is independent of n_1.

25. Let S_n denote the sum of the first n terms of an A.P. If $S_{en} = 3\ S_n$, then prove that $\frac{S_{3n}}{S_n} = 6$.

26. Find the sum of the series : 3 + 5 + 7 + 6 + 9 + 45 + 9 + 13 + 17 + ... to 3n terms.

27. How many terms are there in the A.P. whose first and fifth terms are –14 and 2 respectively and the sum of the terms is 40 ?

28. The sum of first 7 terms of an A.P. is 10 and that of next 7 terms is 17. Find the progression.

29. The number of terms of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

20. The number of terms of an A.P. is even; the sum of odd terms is 24, of the even terms is 30, and the last term exceeds the first by 10½, find the number of terms and the series.

21. If the 5th and 12th terms of an A.P. are 30 and 65 respectively, what is the sum of first 20 terms ?

22. If S_1 be the sum of (2n + 1) terms of an A.P. and S_2 be the sum of its odd terms, then prove that :
$S_1 + S_2 = (2n + 1) : (n + 1)$

23. Three numbers a, b, c are in A.P. iff 2b = a + c.

24. If a^2, b^2, c^2 are in A.P., then prove that the following are also in A.P.

(i) $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$

(ii) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$

25. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., prove that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

26. If a, b, c are in A.P., then prove that :

(i) $(a - c)^2 = 4(b^2 - ac)$

(ii) $a^3 + 4b^3 + c^3 = 3b(a^2 + c^2)$

27. Insert three arithmetic means between 3 and 19.

28. There are n A.M.s between 3 and 17. The ratio of the last mean to the first mean is 3 : 1. Find the value of n.

29. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. Howlong will it take him to clear the loan ?

30. A man arranges to pay off a debt of Rs. 3600 by 40 annual instalments which form an arithmetic series. When 30 of the instalments are paid, he dies leaving one–third of the debt unpaid, find the value of the first instalment.

31. A piece of equipment cost a certain factory Rs. 600000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year, and so on. What will be its value at the end of 10 years, all percentages applying to the original cost ?

EXERCISE-2

Answer each of the following questions in one word or one sentence of as per exact requirement of the questions :

1. Write the common difference of an A.P. the sum of whose first n terms is $\frac{P}{2}n2 + Qn$.
2. If the sums of n terms of two arithmetic progressions are in the ratio $2n + 5 : 3n + 4$, then write the ratio of their mth terms.

EXERCISE-3

Mark the correct alternative in each of the following

1. Sum of all two digit numbers which when divided by 4 yield unity as remainder is
   (a) 1200 (b) 1210 (c) 1250 (d) none of these
2. If S_n denotes the sum of first n terms of an A.P. $< a_n >$ such that $\frac{S_m}{S_n} = \frac{m^2}{n^2}$, then $\frac{a_m}{a_n} =$
   (a) $\frac{2m+1}{2n+1}$ (b) $\frac{2m-1}{2n-1}$ (c) $\frac{m-1}{n-1}$ (d) $\frac{m+1}{n+1}$
3. If the sum of n terms of an A.P. is $2n^2 + 5n$, then its nth term is
   (a) $4n - 3$ (b) $3n - 4$ (c) $4n + 3$ (d) $3n + 4$
4. If $a_1, a_2, a_3, \ldots a_n$ are in A.P. with common difference d, then the sum of the series $\sin d[\sec a_1 \sec_2 + \sec a_2 \sec a_3 + \ldots + \sec a_{n-1} \sec a_n]$, is
   (a) $\sec a_1 - \sec a_n$ (b) $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$ (c) $\cot a_1 - \cot a_n$ (d) $\tan a_1 - \tan a_n$